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PROBLEM OF THE OPTIMUM SKI JUMP

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Depending on the properties of the ski jump, a skier completes his flight in 2.5–4.5 sec and can influence his motion trajectory in the air by changing the angle of attack of the body. Query: How should a sportsman control his body in flight in order to touch down at the greatest distance?

A formulation of this problem and its numerical solution on an electronic computer as an optimum control problem are presented below under the following assumptions: motion of the center of mass of a skier – ski system subjected to gravity, a drag R and a lift Y is considered. The equations of motion and the initial conditions are

$$\frac{dv}{dt} = -\frac{R}{m} - g\sin\theta, \qquad \frac{d\theta}{dt} = \frac{Y}{mv} - \frac{g\cos\theta}{v}$$
$$\frac{dx}{dt} = v\cos\theta, \qquad \frac{dy}{dt} = v\sin\theta$$
$$R = {}^1/_2\,\rho v^2 S c_x, \qquad Y = {}^1/_2\,\rho v^2 S c_y$$
$$t = 0,\ x = 0,\ y = 0,\ v = v_0,\ \theta = \theta_0$$

Here t is the time, x, y, v, θ are the horizontal range, height, modulus of the velocity and slope of the velocity to the x-axis, respectively, m is the system mass, g the acceleration of gravity, ρ the air density, S the characteristic area, and c_x and c_y the aerodynamic coefficients.

The dependence of c_y and c_x on the angle of attack α are taken from [1], where experimental curves obtained as a result of wind tunnel tests on skiers are presented. For convenience in the calculations, these curves have been approximated by the dependencies $c_y(\alpha) = -0.000250\,\alpha^2 + 0.0228\,\alpha - 0.0920$ and $c_x(\alpha) = 0.0103\,\alpha$. The angle of attack can vary between $\alpha_{\min}(t)$ and $\alpha_{\max}(t)$.

The motion is modeled on the profile of the Planitsa (Yugoslavia) jump, on which the absolute world's record jump of 165 m was established (see Fig. 1; a diagram of the angles and forces acting on the skier in flight is given in the upper right corner).

Find the time dependence of the angle of attack $\alpha(t)$ for a maximum jump measured along the hill profile from the take-off table. The problems of the maximum range at the time T of flight termination and of the maximum horizontal range $x(T)$ are evidently equivalent in this case.

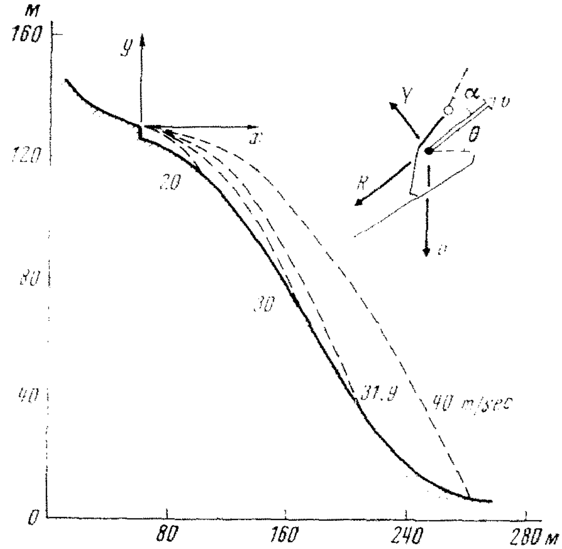

Fig. 1

The formulated variational problem with a free right endpoint and indefinite time differs from the problem in [2] by the form of the dependencies of c_x and c_y on the angle of attack and by the termination conditions of the process. As will be shown, the qualitative time history of the optimal angle of attack is of the same character as in [2].

It is convenient to consider the problem in dimensionless variables by selecting v_0 as the velocity unit, v_0 / g as the time unit, and m as the mass unit.

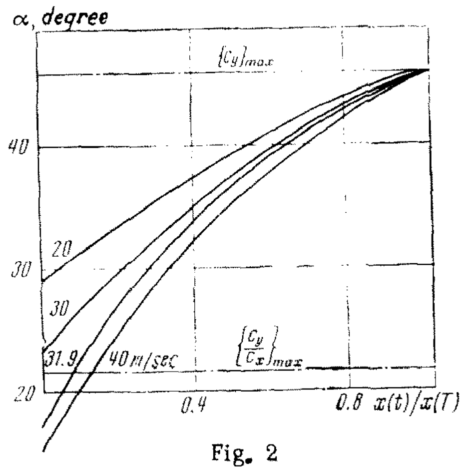

Fig. 2

Therefore, it is necessary to determine that control $\alpha(t)$ which will maximize the functional $x(T)$ at the time T when the skier turns out to be on the landing slope of the jump, i.e. under the condition $h\ (x\ (T),\ y\ (T)) = 0$, where $h\ (x,\ y) = 0$ is the jump profile.

In conformity with the Pontriagin maximum principle, the equations for the conjugate variables, the boundary conditions of the right end, and the selection rule for α which will maximize the Hamiltonian function for any value of the coordinates and momenta, can be written down.

The problem in this form was solved on an electronic computer by using the algorithm from [3] for four values of the initial velocity $v_0 = 20, 30, 31.9$ and 40 m/sec at a separation angle $\theta_0 = 0.0902$. In conformity with the experimental data, the following values of the parameters were taken

$$\frac{S}{mg} = 0.01\,\frac{M^2}{\text{kg}}, \quad \rho = 0.125\,\frac{\text{kg sec}^2}{M^4}, \quad \alpha_{\min} = 5^\circ, \quad \alpha_{\max} = 80^\circ$$

Presented in Fig. 2 are dependencies of the optimal controls $\alpha^\circ\ (t)$ for the initial motion velocities listed, to which flight ranges of 39, 113, 174 and 245 m correspond. A third modification of the optimal trajectory is selected according to the initial conditions under which the record 165 m jump was performed. However, this does not yield the right to far-reaching conclusions since the aerodynamic characteristics of the record maker are not known.

It should be noted that up to now it has been considered in sports [1] that the farthest jump is achieved upon conserving a constant angle of attack corresponding to the maximum aerodynamic capacity (*). In this case, the flight trajectory is 13 m less than the optimal for a 30 m/sec initial velocity as the computations show. Under the condition of conserving the angle of attack with maximum value of the lift coefficient, the range of the jump is only 10 m less than the length of the optimal trajectory for the same initial conditions. The trajectories for the four modifications mentioned are shown in Fig. 1. The profile of the Planitsa jump is deformed (stretched out double along the vertical, see the grid scale) for clarity.

The graphs presented permit the following deductions to be made. If the skier should take a position with a slight angle of inclination which would assure negligible frontal drag on the initial part of the flight, then the body angle of attack on the second half of the flight should gradually approach the angle with the maximum value of the lift coefficient, i.e. the preference in the final flight phase is given to gliding quality to the detriment of air drag. In flights at a 20–25 m/sec velocity, which is intrinsic to medium jumps, the frontal drag factor plays a lesser part as compared to jumps from bigger hills (28–33 m/sec initial velocity). Hence, for jumps from medium hills, jumps with a sufficiently high angle of attack (around 30°) in the first phase of the motion yield the optimal trajectories, while flights with a low angle of attack at the begin ning of the flight (from 15–23°) yield the best results for jumps from long hills.

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*) Editor's Note. This term is expressed by the symbol $c_y\ /\ c_x$, and in English nomenclature corresponds to lift-drag ratio C_L / C_D.

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